

# COUPLING OF DIELECTRIC RESONATORS TO RECTANGULAR WAVEGUIDES\*

Kawthar A. Zaki and Chunming Chen

University of Maryland, Department of Electrical Engineering  
College Park, MD 20742

## Summary

A method for calculation of the coupling of a dielectric loaded resonator to a rectangular waveguide by means of a long thin slot is presented. Experimental measurements performed on several cases are compared to the calculations. Effect of the resonator's field variation on the coupling calculations are discussed. It is shown that the small aperture theory fails to predict the coupling coefficient variation as a function of the slot length. The actual (measured) variation is predicted properly by taking into account the field distribution in the resonator.

## I-INTRODUCTION

This paper reports on the calculations and measurements of the coupling of a dielectric loaded resonator to a rectangular waveguide by means of a long thin slot, as shown in Fig. 1. This type of structure is encountered in microwave and millimeter wave applications such as filters [1], or oscillators [2], when a waveguide must be used as the transmission medium. The resonant frequency and fields in the resonators are computed using the method of reference [3], while the coupling coefficient (or external  $Q$ , i.e.  $Q_e$ ) is estimated based on a modification to the small aperture theory [4]. Certain limitations of the usefulness of the small aperture theory for the accurate calculation of the coupling coefficient of relatively long slots are discussed. These limitations are shown by calculations of the field distribution in the resonator [3], and then verified experimentally. Both the theoretical and experimentally measured results are presented for two different resonant modes, with good agreement.

## II-THEORY

The structure under consideration is shown in Figure 1. The rectangular waveguide of width  $A$  and height  $B$  is assumed to support the normal  $TE_{10}$  mode. The dielectric loaded resonator consists of a dielectric disk of length  $\ell$ , radius  $a$  and relative dielectric constant  $\epsilon_r$ , symmetrically placed inside a perfectly conducting cylindrical enclosure

of length  $L$  and radius  $b$ . The waveguide is coupled to the resonator through a centrally located slot of width  $w$  and length  $d$  in the end plane of the resonator of thickness  $t$ .

The coupling between the waveguide and the cavity is expressed by [4] the external quality factor,  $Q_e$  as:

$$Q_e = \frac{\omega U}{P_L} \quad (1)$$

where:

$\omega$  is the angular resonant frequency of the cavity

$U$  is the energy stored in the cavity, and

$P_L$  is the power loss through the iris, given by:

$$P_L = \frac{A_a^2 S_a}{2} \quad (2)$$

where

$S_a$  is the peak power of the Poynting vector fields in the waveguide, and  $A_a$  is the amplitude of the normal mode fields excited in the waveguide by the cavity fields. In the small aperture theory [4]  $A_a$  is expressed as:

$$A_a = \frac{\omega \mu_o M H H_1}{S_a} \quad (3)$$

where  $M$  is the magnetic polarizability of the iris,  $H$  and  $H_1$  are the amplitudes of the tangential magnetic fields in the waveguide and the resonator at the center of the iris respectively. Equation (3) does not reflect the effect of the field variation along the coupling iris, and hence will not yield the correct coupling if the slot is large or the fields have large variation across the slot. To account for these effects Cohen [5] introduced empirical resonance and thickness correction factors in the polarizability of the slot. To take the effect of rapid field variations into account in this paper equation (3) is modified to:

$$A_a = \frac{\omega \mu_o M}{S_a} \frac{1}{wd} \int_{\text{iris area}} \bar{H}_{\text{guide}} \cdot \bar{H}_{\text{cavity}} da \quad (4)$$

The waveguide  $TE_{10}$  mode fields are expressible as:

$$\bar{E}_{\text{guide}} = H_o \sqrt{\frac{\mu_o}{\epsilon_o}} \frac{\lambda_g}{\lambda_o} \cos \frac{\pi x}{A} e^{j(\omega t - \beta z)} \hat{a}_y \quad (5-a)$$

\* This material is based upon work supported by the National Science Foundation under Grant No. ECS-8320249.

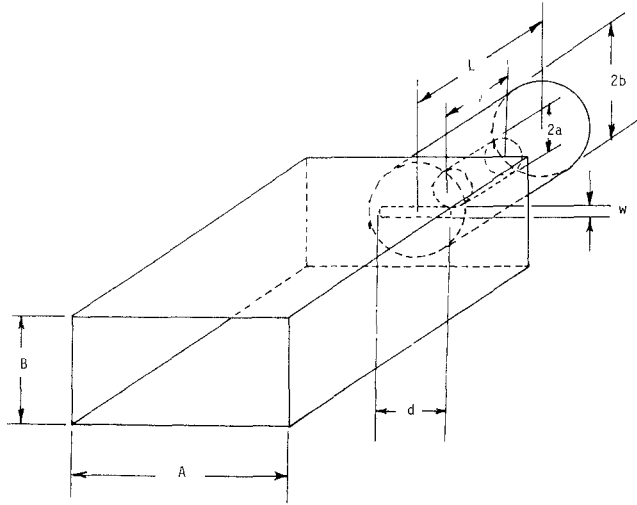


Figure 1 - Dielectric Loaded Resonator coupled to a rectangular waveguide.

$$\vec{H}_{\text{guide}} = H_0 \left[ \cos \frac{\pi x}{A} \hat{a}_x - j \frac{\lambda_g}{2A} \sin \frac{\pi x}{A} \hat{a}_z \right] e^{j(\omega t - \beta z)} \quad (5-b)$$

From these expressions, the peak power  $S_a$  is computed as:

$$\begin{aligned} S_a &= \int_{\text{guide cross section}} \vec{E}_{\text{guide}} \times \vec{H}_{\text{guide}} \cdot d\vec{s} \\ &= \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{k_0}{\beta} \frac{AB}{2} H_0^2 \end{aligned} \quad (6)$$

The dielectric resonator fields and resonant frequency are computed as described in reference [3]. From these fields the tangential magnetic field  $\vec{H}_{\text{guide}}$  at the cavity wall used in equation (4) is known; and closed form expressions for the energy stored in the resonator  $U$  is computed from:

$$U = \frac{\epsilon}{2} \int_{\text{cavity volume}} |\vec{E}_{\text{cavity}}|^2 dV$$

Thus all quantities necessary for the calculation of  $Q_e$  in equation (1) are known.

### III-Results

Variation of the magnetic field intensity in the end plane as a function of  $r$  for typical dielectric loaded resonators of the lowest resonant nonaxially symmetric mode ( $HEH_{11}$ ) are shown in Fig. 2. Coupling through the slot of Figure 1 is due to the magnetic fields  $H_x$  in the waveguide and  $H_r$  (at  $\theta = 0, \pi$ ) in the resonator. As seen in Figure 2 variation of the magnetic field  $H_r$  along the slot length depends on the ratio  $(L/\ell)$  and the length of the slot. The radial magnetic fields of this mode have a maximum at  $r = 0$  and vanish at  $r = b$  (as required by the boundary conditions). Between  $r=0$  and  $r = b$ , the field variation depends on the ratio  $(L/\ell)$ . For  $(L/\ell)$  close to 1,  $H_r$  decreases monotonically from its maximum at  $r=0$  to zero at  $r = b$ . As  $(L/\ell)$  is increased  $H_r$  starts to have zeros in the range  $0 < r < b$ , and reverses its direction over parts of that range. This effect is maximized in Fig.

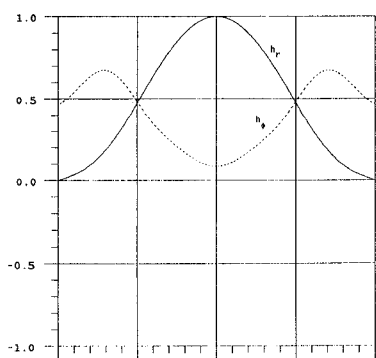
2-c for  $(L/\ell) = 1.2$  where the zeros of  $H_r$  appear to be closer to the center. For this case, if the length of the coupling slot  $d$  is increased from a small value to a value less than the position of the zero of  $H_r$ , it is expected that the coupling due to that slot increases. Further increase in the slot length beyond the zero of  $H_r$  causes the magnetic fields in the resonator and the waveguide to have opposite directions, and hence the rate of increase of the coupling value should decrease. The variation of the external quality factor,  $Q_e$  with the slot length  $d$  for two different slot widths is shown in Figure 3, for the resonator parameters of Figure 2-c. Experimentally measured points are shown in Figure 3 for both slot widths of .036 inches and .05 inches by stars and squares respectively. The waveguide used in the experimental measurements is WR-187, and the measured resonant frequency varied between 4.48 and 4.416. The computed resonant frequency of the unperturbed resonator is 4.623 GHz. Variation of the magnetic field intensity in the end plane of a dielectric loaded resonator of the second lowest resonant non axially symmetric mode ( $HEE_{11}$ ) are shown in Figure 4. Variation of the external quality factor  $Q_e$  with the slot length  $d$  for two different slot widths is shown in Fig. 5. The coupling for this mode increases monotonically since the resonator field is almost constant across the slot.

### IV-Conclusion

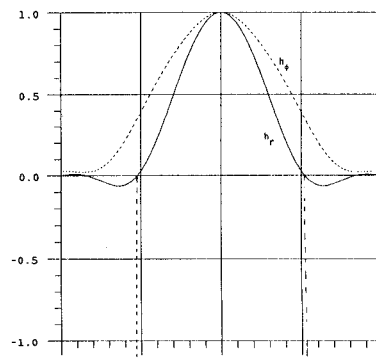
The simple modification introduced in the small aperture coupling theory allows for the rapid resonator's field variations along the length of the slot to be accounted for. Results of the measurements and the calculations confirms the validity of this modification for two sets of the first lowest non axially symmetric resonant modes.

### References

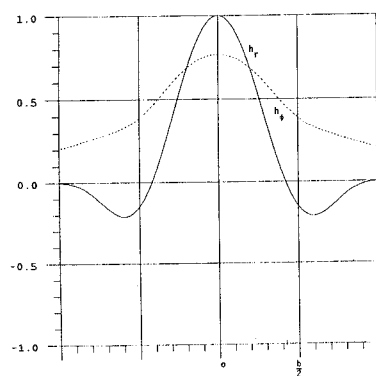
- [1] S.J. Fiedzinski, "Dual-mode dielectric resonator loaded cavity filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-30, pp. 1311-1316, September 1982.
- [2] J. Abe et al., "A highly stabilized low noise GaAs FET integrated oscillator with a dielectric resonator in the c-band," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-26, pp. 156-163, March 1978.
- [3] K.A. Zaki and A.E. Atia, "Modes in dielectric-loaded waveguides and resonators," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-31, No. 12, December 1983, pp. 1039-1045.
- [4] Matthaei, Young and Jones, "Microwave filters, impedance - matching networks, and coupling structures, McGraw-Hill, 1964.
- [5] S.B. Cohn, "Microwave coupling by large apertures," *Proc. IRE* 40, pp. 696-699, June 1952.



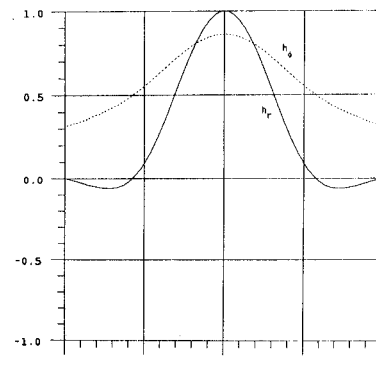
(a)  $(L/\ell)=1.01$



(b)  $(L/\ell)=1.1$



(c)  $(L/\ell)=1.2$



(d)  $(L/\ell)=2$

Figure 2 - Magnetic Field Intensity  $h_r$  versus  $r$  for  $HEH_{11}$  mode

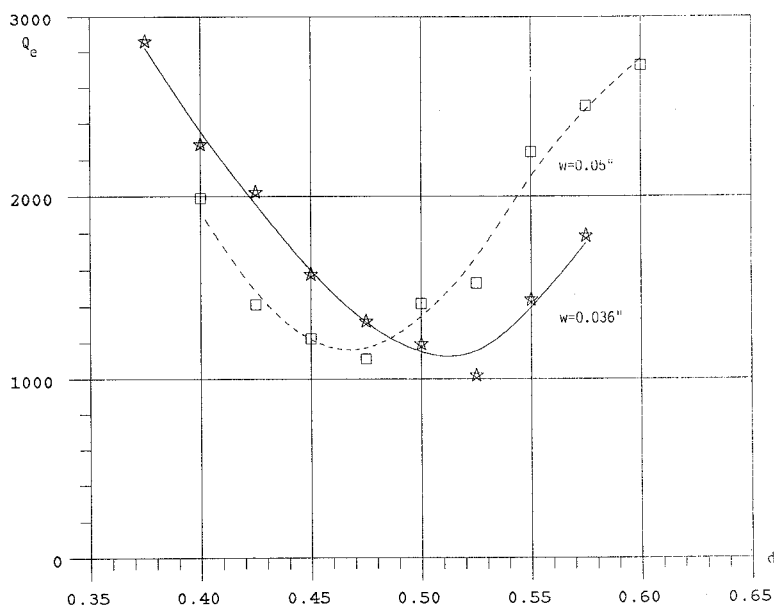


Figure 3 - External Quality Factor  $Q_e$  versus slot length  $d$  for  $HEH_{11}$  mode

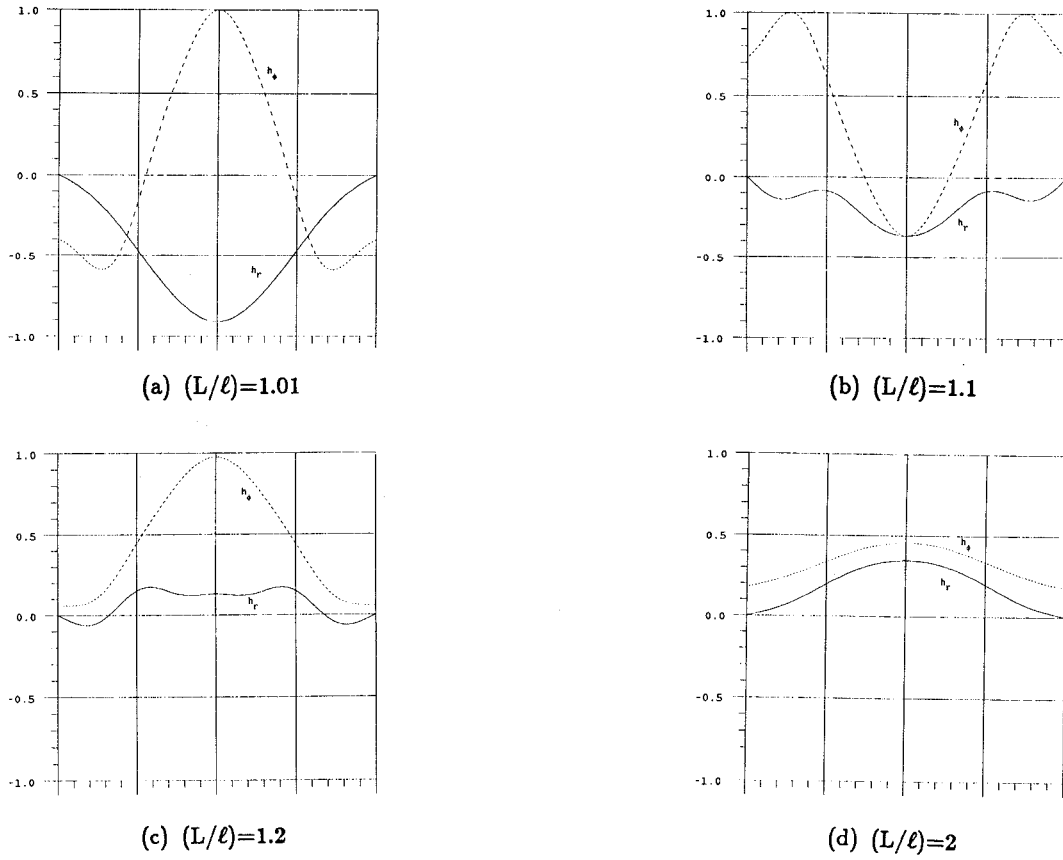


Figure 4 - Magnetic Field Intensity  $h_r$  versus  $r$  for  $HEE_{11}$  mode

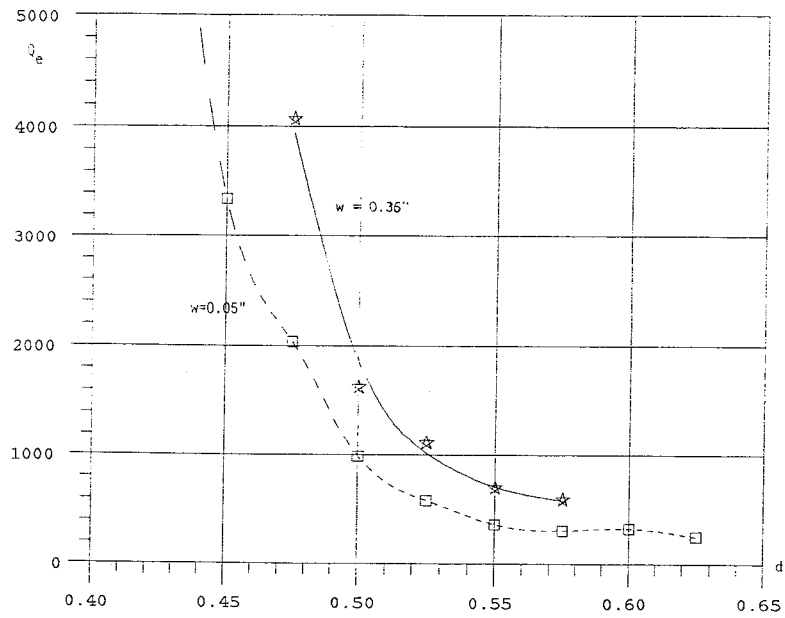


Figure 5 - External Quality Factor  $Q_e$  versus slot length  $d$  for  $HEE_{11}$  mode